# Correlation of the angular dependence of spin-transfer torque and giant magnetoresistance in the limit of diffusive transport in spin valves

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Angular variation in giant magnetoresistance and spin-transfer torque in metallic spin-valve heterostructures is analyzed theoretically in the limit of diffusive transport. It is shown that the spin-transfer torque in asymmetric spin valves can vanish in noncollinear magnetic configurations, and such a nonstandard behavior of the torque is generally associated with a nonmonotonic angular dependence of the giant magnetoresistance, with a global minimum at a noncollinear magnetic configuration.

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#### I. INTRODUCTION

The macroscopic model of electronic transport in magnetic metallic multilayers, developed by Valet and Fert,<sup>1</sup> is commonly used for interpretation of experimental results on current-perpendicular-to-plane (CPP) giant magnetoresistance (GMR). The model includes several phenomenological parameters which can be extracted from fitting experimental and theoretical results on resistance/magnetoresistance in collinear (parallel and antiparallel) magnetic configurations. The model is based on the assumption that the spin-diffusion length is longer than mean-free path, and the latter is smaller than the layer thickness. Recently, Penn and Stiles<sup>2</sup> showed that the Valet-Fert model<sup>1</sup> is justified even for spin-diffusion lengths comparable to the mean-free paths. Moreover, the model fits well to experimental results even when the meanfree paths are comparable to the layer thicknesses.

Validity of the model, however, can be justified only when it gives results which are in agreement with experiment not only in the collinear configurations (too many fitting parameters) but also in the full range of angles between magnetic moments of the layers' magnetizations. In a recent paper<sup>3</sup> the Valet-Fert model<sup>1</sup> has been extended to noncollinear configurations, but only the angular variation in the spin-transfer torque (STT) was analyzed there. The spin torque plays a crucial role in the phenomenon of current-induced magnetic switching (CIMS). It turned out that both CPP-GMR and CIMS effects are correlated. Moreover, there are normal and inverse GMR and also normal and inverse CIMS phenomena. Accordingly, four different possibilities can be found in real systems.<sup>3</sup> Indeed, the normal and inverse CPP-GMR and/or CIMS have been demonstrated by manipulating the bulk and/or interface spin-asymmetry parameters.<sup>4</sup>

It has been shown in Ref. 3 that the STT in asymmetric structures can vanish at a noncollinear configuration, which has a significant impact on the stability of magnetic configuration. As a result, precessional states in zero magnetic field were predicted in the Co/Cu/Py nanopillars<sup>5</sup> and later experimentally confirmed.<sup>6</sup> One may naturally expect that the non-standard behavior of STT may be associated with some anomalous angular behavior of the CPP-GMR. This problem is addressed here, and we show that a global minimum in resistance of asymmetric structures may occur in a noncol-

linear configuration. This nonmonotonic behavior of the resistance (and consequently also GMR) is generally accompanied by the nonstandard angular dependence of STT. These two features seem to be characteristic of asymmetric systems in the diffuse transport regime, provided the system's parameters obey some conditions.

### **II. MODEL OF CPP-GMR**

Within the diffusive approach,<sup>1,3,7</sup> spatial dependence of the average electrochemical potential in a ferromagnetic (F) layer has the general form:  $\bar{\mu} = -\beta g + Cx + G$ , where the axis *x* is normal to the structure and *g* is the spin accumulation, *g* =  $A \exp(x/l_{sf}) + B \exp(-x/l_{sf})$ , with  $l_{sf}$  being the spindiffusion length. Similar formula also holds for normal-metal (N) layers, but with  $\beta = 0$ . All the constants (*A*, *B*, *C*, and others) entering the general solution of the diffusive equations in different layers can be determined from the corresponding boundary conditions.<sup>1,3,7</sup>

The driving field can be then calculated as  $E(x) = (1/e)(\partial \overline{\mu}/\partial x)$ .<sup>1</sup> The presence of N/F interfaces gives rise to additional voltage drops due to spin accumulation in their vicinity. The total voltage drop can be then written as  $\Delta V = \sum_i \Delta V_i$ , where  $\Delta V_i$  is the voltage drop in the *i*th layer of the spin valve (voltage drops at interface resistances will be included to the ferromagnetic layers). When the index *i* corresponds to a ferromagnetic layer,  $\Delta V_i = \Delta V_i^{\text{SI}} + \Delta V_i^{\text{spl}}$ . If, however, *i* corresponds to a nonmagnetic layer,  $\Delta V_i = \Delta V_i^{\text{SI}} + \rho_i d_i I_0$ , where  $\rho_i$  is the bulk resistivity of the normal metal,  $d_i$  is the corresponding layer thickness, and  $I_0$  is the current density. The voltage drops due to spin accumulation (in magnetic and nonmagnetic layers) read

$$\Delta V_i^{\rm SI} = \int_{x \in d_i} [E(x) - E_0] dx, \qquad (1)$$

where the corresponding electric field  $E_0$  is taken far from the interface. Apart from this,  $\Delta V_i^{\text{spl}} = I_0 R_i^{\text{spl}}$  for ferromagnetic films,  $R_i^{\text{spl}} = [(1/R_{i\uparrow}) + (1/R_{i\downarrow})]^{-1}$ , and  $R_{i\sigma} = R_{i\sigma}^L + d\rho_{i\sigma} + R_{i\sigma}^R$  (for  $\sigma = \uparrow, \downarrow$ ), with  $\rho_{i\sigma}$  being the corresponding spin dependent bulk resistivity and  $R_{i\sigma}^L (R_{i\sigma}^R)$  denoting the interfacial resistances (per unit square) associated with the left (right) interface of the *i*th (ferromagnetic) film. The total resistance of the system (per unit square) is  $R = \Delta V/I_0$ , while the magnetoresistance,  $\Delta R(\theta) = R(\theta) - R(0)$ , describes a change in the total system resistance when magnetic configuration varies from a noncollinear to parallel one. We note that what one needs to calculate are the  $\Delta V_i^{\text{SI}}$  contributions only. It is convenient to define reduced magnetoresistance as

$$r(\theta) = \frac{R(\theta) - R_{\rm P}}{R_{\rm AP} - R_{\rm P}}.$$
(2)

Several theoretical approaches have been proposed to describe the angular variation in GMR.<sup>8</sup> The explicit form for the reduced magnetoresistance,  $r(\theta) = \sin^2(\theta/2)/[1 + \chi \cos^2(\theta/2)]$ , has been extracted within the magnetocircuit theory<sup>9</sup> and diffusive approach.<sup>10</sup> In recent measurements on Py/Cu/Py valves<sup>11</sup> the parameter  $\chi$  has been treated as a fitting parameter, and the  $r(\theta)$  dependence was found to describe the experimental data relatively well. However, this formula breaks down for asymmetric spin valves, where a global minimum of the system resistance may appear at a noncollinear configuration.<sup>11</sup>

## **III. RESULTS**

The minimum in resistance (and also in GMR) in a noncollinear configuration appears only in asymmetric spin valves. In the following we discuss the angular dependence of the STT and GMR in spin valves for positive current density,  $I_0 > 0$  (electrons flow from right to left or in other words charge current flows from the layer of thickness  $d_1$  to the layer of thickness  $d_2$ ). Figure 1(a) shows electric-field profile in the  $Co(d_1)Cu(10)Co(8)$  spin valve for  $d_1=16$  nm sandwiched between semi-infinite Cu leads and for both parallel (P) and antiparallel (AP) magnetic configurations. The angular dependence of the corresponding voltage drops within the Co layers is shown in Fig. 1(b). The voltage drop within the Co(16) layer exhibits a very weak minimum for  $\theta \simeq \pi/3$ . The minimum becomes much more pronounced for larger layer thickness, as shown in Fig. 1(c) for  $d_1=60$  nm. Since the total voltage drop is a sum of all drops in the individual layers, the GMR can exhibit a minimum at a noncollinear configuration when the resistance decrease in the thick F layer overcomes the resistance increase in the thin F layer. The global minimum arises as a result of the spatial depletion of electrical field in the thick F layer, which is a consequence of spin accumulation discontinuity at the N/F interface controlled by the mixing conductances. The reduced GMR,  $r(\theta)$ , is shown in Fig. 1(d) for both values of  $d_1$ . The nonmonotonic behavior of the reduced GMR is more pronounced in spin valves that are more asymmetric [see Fig. 1(e)].

In Figs. 2(a)-2(c), the diagrams present the regions of layer thicknesses, where the nonmonotonic behavior of the reduced GMR can be observed (gray regions). For the Co/Cu/Co spin valves [Fig. 2(a)] as well for the Py/Cu/Py ones [Fig. 2(b)], the diagrams are symmetric with respect to  $d_1 = d_2$ , and the nonmonotonic angular variation in the GMR (global minimum at a noncollinear configuration) can be noticed for spin valves with significantly different layer thick-



FIG. 1. (Color online) Transport characteristics of the biased  $Cu/Co(d_1)/Cu(10)/Co(8)/Cu$  spin valve. (a) Spatial dependence of the electric field in the system for  $d_1=16$  nm and for P and AP configurations. Angular dependence of the voltage drops in  $Co(d_1)$  shown by dashed (blue) line and Co(8) by solid (red) line for (b)  $d_1=16$  nm and (c)  $d_1=60$  nm. (d) Angular dependence of the reduced magnetoresistance. (e) Reduced magnetoresistance as a function of  $\theta$  and  $d_1$ . In parts (a)–(c) the current density  $I_0 = 10^8$  A/cm<sup>2</sup> was assumed. The other parameters are as in Ref. 3.

nesses. In Co/Cu/Py spin valves, where an additional asymmetry appears due to different magnetic materials, a nonmonotonic angular variation in the GMR can be observed even for comparable layer thicknesses [see Fig. 2(c)]. This is mainly due to strong asymmetry in spin-diffusion lengths of Co and Py, but difference in the bulk as well as interface spin asymmetries of the Co and Py also contributes to the nonmonotonic behavior.

Experimental observations on Py/Cu/Py spin valves revealed a weak nonmonotonic angular variation in the GMR effect.<sup>11</sup> This has been attributed to the absorption of transverse spin accumulation in a noncollinear configuration, which reduces the resistance. Such absorption also gives rise to the STT acting on the F layer,<sup>12</sup> which in asymmetric spin valves can exhibit an anomalous (nonstandard) angular dependence.<sup>5,6</sup> In systems with a nonstandard STT, the transverse component of spin current (accumulation) at the active N/F interface vanishes at a certain noncollinear configuration. The presence of a GMR minimum in a noncollinear configuration can be thus related to the nonstandard STT.



FIG. 2. (Color online) Diagrams illustrating presence of a global magnetoresistance minimum at noncollinear configurations—gray regions—and angular spin-transfer torque dependences. (a) Diagram for the Co( $d_1$ )Cu(10)Co( $d_2$ ), (b) Py( $d_1$ )Cu(10)Py( $d_2$ ), and (c) Co( $d_1$ )Cu(10)Py( $d_2$ ) spin valves. The solid (red) and dashed (blue) lines denote critical thicknesses where  $\partial \tau / \partial \theta|_{\theta \to 0} = 0$  for the torque exerted on the left F layer of thickness  $d_1$  and right layer of thickness  $d_2$ , respectively. [(d)–(f)] Angular dependence of the spin-transfer torques acting on the left F layer of thickness  $d_2$ —shown by solid (red) lines—and right F layer of thickness  $d_2$ —shown by dashed (blue) lines—for systems corresponding to the dots in the left panel [(a)–(c)].

The STT appears due to absorption of the transverse spin current component  $j_{\perp}$  at the N/F interface<sup>12</sup> and can be calculated as

$$\boldsymbol{\tau} = \frac{\hbar}{2} (\boldsymbol{j}_{\perp}^{L} - \boldsymbol{j}_{\perp}^{R}), \qquad (3)$$

where the superscripts L and R denote the left and right interfaces, respectively, associated with the F layer. Dependence of the STT can also be expressed explicitly in terms of the mixing conductances and spin accumulation at the normal-metal side of the N/F interface.<sup>3,5</sup> The STT consists generally of both in-plane and out-of-plane components. Since the latter component is much smaller than the former one (due to small imaginary part of the mixing conductances<sup>13</sup>), in the following discussion we will consider only the in-plane component. In asymmetric spin valves, the proper choice of magnetic materials and/or layer thicknesses can result in vanishing STT at a noncollinear magnetic configuration.<sup>5</sup> Such a nonstandard STT destabilizes both collinear configurations for positive current and stabilizes both configurations for negative current.<sup>3,5</sup> The former case is of particular interest as the nonstandard torque leads to current-induced steady-state oscillations in the absence of external magnetic field.<sup>5,6</sup> In Fig. 2(d) we show the angular dependence of STT in the Co(60)Cu(10)Co(8) spin valve exerted on the Co(60) (solid line) and Co(8) (dashed line). The STT acting on the Co(8) layer destabilizes P and stabilizes AP configurations, whereas the torque acting on the Co(60) vanishes at a noncollinear configuration and stabilizes both P and AP configurations. The torques in Fig. 2(d) correspond to the system indicated by the dot in Fig. 2(a). This point is below the critical line, given by  $\partial \tau / \partial \theta |_{\theta \to 0} = 0$ , which identifies the region where a nonstandard STT acting on the  $Co(d_1)$  layer appears. When the layer thicknesses are above the critical line, but still in the gray region [see the dot in Fig. 2(b)], the torque acting on the particular F layer vanishes only for the collinear configurations, as shown in Fig. 2(e) for the Py(1)Cu(10)Py(6) spin valve, but reduced GMR still exhibits a global minimum for a noncollinear configuration. Since the critical lines are close to the boundary of the nonmonotonic angular GMR behavior (gray regions), the nonstandard STT is correlated with the nonmonotonic angular variation in GMR.

At the critical angle  $\theta_c$ , where the torque  $\tau$  vanishes, the transverse component of spin accumulation at the active in-



FIG. 3. (Color online) Parametric plots of the in-plane spin accumulation components and angular dependences of the angle  $\theta_g$ between the spin accumulation and spin moments. (a) Spin accumulation at the left (F/N) interface; (b) spin accumulation at the right (N/F) interface at the normal-metal side (N) in the vicinity of the active interfaces. The spin accumulation components are expressed in their local reference frames. The dashed (red) lines correspond to Co(60)Cu(10)Co(8); solid (black) lines to Py(1)Cu(10)Py(6); and dotted (blue) lines to Co(8)Cu(10)Py(8). The filled dots correspond to the parallel configuration of the layer magnetizations. Angular dependence of the  $\theta_g$  at the (c) left F/N and (d) right N/F interfaces.

terface disappears. In a general case, however, the angle  $\theta_o$ between the spin moment of the F layer and spin accumulation vector at the normal-metal side of the N/F interface is nonzero. Angular dependence of the STT can be then expressed as a function of  $\theta_{a}$ .<sup>3</sup> In Figs. 3(a) and 3(b) the inplane spin accumulation components at the normal-metal side (in the spacer layer) at the left and right interfaces are shown for the spin valves considered in Figs. 2(d)-2(f). The components are expressed in local coordinate frames, where  $g_z$  points along magnetization in the left F layer whereas  $g'_z$ along magnetization in the right F layer. The STT acting on Co(8) in the Co(8)Cu(10)Py(8) spin valve exhibits regular behavior. Spin accumulation in the P configuration is positive and has comparable amplitudes in the vicinity of both interfaces [see the dots on the dotted lines in Figs. 3(a) and 3(b)] due to long spin-flip length in Cu  $(l_{sf} \simeq 1 \ \mu m)$ . When magnetization of the right layer rotates in the film plane, spin accumulation at the left interface roughly follows the net spin of the Py(8) layer, and the angle  $\theta_g$  is a monotonic function of  $\theta$  [see Fig. 3(c)]. Figure 2(f) shows that STT exerted on the Py(8) layer vanishes in a noncollinear configuration ( $\theta = \theta_c$ ), for which the  $g'_v$  component also vanishes. At  $\theta = \theta_c$  one finds  $\theta_g = 0$ . For the  $\dot{C}o(60)Cu(10)Co(8)$  system, STT acting on the Co(60) layer shows a nonstandard behavior which is qualitatively similar to that for the Py(8) layer in the Co(8)Cu(10)Py(8) spin valve. Angular variation in STT for the Py(1)Cu(10)Py(6) spin valve, shown in Fig. 2(e), vanishes regularly in P and AP configurations, and  $\theta_a$  is a monotonic function of  $\theta$ .

Noncollinear configuration of the F layer magnetizations leads to discontinuities of the spin accumulation at the F/N interfaces [angle  $\theta_g$  in Figs. 3(c) and 3(d)]. From this we deduce that if one takes the thickness of one of the F layers smaller than the corresponding spin-diffusion length and thickness for the second F layer is larger than the appropriate spin-diffusion length, then the spin accumulation is predominately determined by the later F layer. One finds then non-standard STT and nonmonotonic GMR angular behavior. We have found that this behavior is mostly controlled by the mixing conductance of the interface between spacer layer and that F layer whose thickness is smaller than the corresponding spin-diffusion length. For instance, reducing the mixing conductance at the Co(8)/Cu(10) interface in the Co(8)Cu(10)Py(8) valve by about 50% lifts the nonstandard STT and GMR behavior.

In conclusion, what stem from the above results are a need for further experimental investigations and that Co/Cu/Py system is a good candidate to test the theoretical predictions. This also could answer the question whether the diffusive approach used to analyze CPP-GMR in collinear configurations is well justified. To arrive at more convincing conclusions one also should correlate the results on GMR with those on STT.

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